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Identification of the Fuel-Thrust Dynamics of a Gas Turbo Engine

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Abstract—**The dynamical model of the fuel-rate to thrust of a laboratory turbo reactor engine is experimentally identified. By applying recursive least squares identification several linear models corresponding to various operating conditions are obtained. In addition, the analysis of the characteristics of the engine shows the presence of a hysteresis loop. These models are combined in order to form a nonlinear dynamical model of the process. The resulting nonlinear model captures the dynamics over the whole operating range of the engine. The results are validated through real time experimental tests. Although there are several reports on gas-turbo-engine identification, models of the fuel-ratio to thrust dynamics are not commonly reported.**

I. Introduction

High performance aerospace turbo engines demand the introduction of several control loops in the engine sub-systems. As is pointed out in [1], the multi-mission requirements of new aircraft demand a significant increase of the propulsion system capabilities. As a result gas turbine-engines are nowadays more complex with corresponding increases in the complexity of the control systems [1]. In order to design adequate control systems for the constant demand in performance, reliable models are needed. This aspect has taken additional relevance together with the development of turbo-engines control systems.

Turbo-engines identification has been a key factor for the improvement of their performance and capabilities under closed loop schemes. In practical applications the most effective and used models for control design are based on frequency domain transfer functions [2]. Nowadays, more sophisticated models have been proposed. For instance, a model for active surge control of an engine is presented and validated with experiments in [3]. In this article lump parameters are introduced in order to define a non-linear dynamical model. A similar approach relying on the thermodynamic nature of the process is presented in [4]. An interesting contribution to the subject is reported in [5], in which several identification techniques are applied in the search of efficiency and cost effectiveness. In [5] engine identification using only ambient-noise data, multi-sine testing, frequency domain identification, least-squares with optimal smoothing and multi-objective genetic programming methods are reported.

The availability of high computational power allows the application of genetic programming and fuzzy logic techniques, as reported in [6,7,8].

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The relevance of accurate modeling is still fundamental, even for control system designs robust to model uncertainty [9].

Most of the identification works reported aim at finding a dynamical representation which considers the fuel rate as input and the shaft velocity as output. In occasions, the pressure and temperature changes along the engine working cycle are also reported. The main reason for the lack of thrust models is that, although the major objective of the turbo engine is to produce thrust, it is not easy to measure this variable during flight. Therefore, most of the models are intended to determine the internal variables accurately in order to obtain adequate estimations of the engine thrust.

The aim of this paper is to identify a dynamical model which considers the fuel-rate as input and the engine thrust as output. For this purpose the laboratory gas turbine engine developed by Turbine Technologies LTD model ML-401 is used. The set-up of this engine allows measuring the fuel-rate and the engine thrust through a load cell system. In addition to the shaft revolution measurement, this platform also allows the measurement of several other variables such as the temperatures and pressures at the: compressor inlet, compressor exit, turbine stage inlet, turbine stage exit and engine exit. However, the present work focuses on the flow-rate to thrust dynamical model since this relationship is not commonly addressed in the literature and may be used in order to validate the thermodynamic internal model of the engine.

A set of experiments at various operating conditions were performed and the data collected saved for analysis. With these data a typical recursive least squares algorithm was applied off-line. The zero/pole structure of the system was elucidated by using the quadratic error and the process bandwidth characteristics. In addition, a characterization of the static flow-rate to thrust relationship was made. This characterization resulted on the identification of a nonlinear hysteresis loop. The final model is a mixture of the mean estimated linear dynamics and the mean nonlinear static gain.

Finally, the model was experimentally validated. This validation shows that the model reflects the main characteristics of the engine and could be used for either closed loop controller design or to predict the effective thrust by measuring only the fuel-flow. The success of the modeling procedure even in the presence of a high level of measuring noise, due to electrical noise and the engine vibrations, suggests that this approach may be used for other turbo-gas engines with simple and low-cost measuring devices.

II. A Laboratory Turbo Reactor Engine

The turbine-gas-engine used in this study is the model SR-30 produced by Turbine Technologies Ltd. The engine includes various pressure and temperature sensors, a load-cell for thrust measurement, a custom winding for reading RPM, and a fuel-flow-rate measurement. The maximum nominal thrust is 89 N at 87000 RPM. At this regime the device ingests 0.5 kg/s of air. The length of the engine is 0.273 m and the exit exhaust diameter is 5.715 cm.

The engine includes pressure transducers in the compressor inlet and exit, in the combustion chamber, in the turbine exit, and in the thrust nozzle exit. It is build up with K-type thermocouples in the compressor inlet and exit; in the turbine inlet; and exit and in the thrust nozzle exit.

Data acquisition is available through a PCI 4351, A/D board with 24 bit resolution for 16 analog inputs with a 60 samples/s capability and a Virtual Bench Logger data acquisition program for monitoring the measured variables on a PC. It is important to note that the actual practical maximum sampling frequency of this system for the selected variables is 13.15hz since the system operates with a multiplexing data acquisition scheme. This figure is important since the vibrations and electrical induced on the load-cell are of much higher frequency. This produces an ill-posed noise problem in the measured thrust signal. Although it is technically possible to use an A/D interface with a better sampling rate, this would not be cost-effective. Therefore, in this article a procedure to reduce the effect of high-frequency noise and vibrations, other than the use of a better A/D interface, is introduced.

Starting the engine requires an external source of high-pressure air at 689 kPa to spin-up the rotor to 10000 RPM. Thereafter, fuel injection and ignition starts the engine. Fuel injection is controlled manually through a lever, which adjusts a valve constricting the fuel flow to the engine.

III. STATICS

The typical operating condition for a gas turbine is to operate in a steady regime. Therefore, accurate prediction of the total thrust in such conditions is important. In order to elucidate the steady state gain of the engine, the input fuel flow was varied in slow stair sequences as the one shown in Fig. 1 and the produced thrust was measured using the load cell (Fig. 2).

The mean steady state thrust for each input level of figures 1 and 2 was calculated. A graphical representation of the relationship between the measured average static trust and the measured fuel flow rate is shown in Fig. [3].

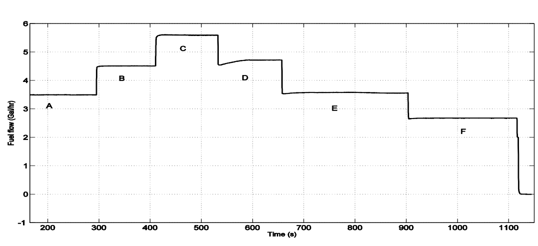

Fig. 1. Experimental fuel flow input

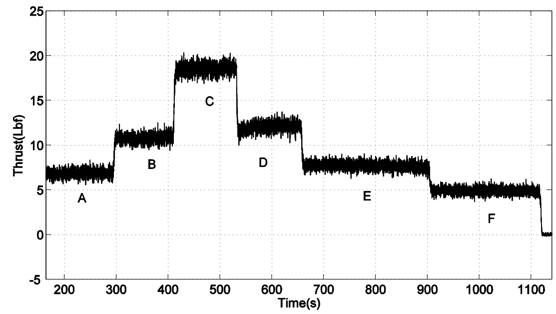

Fig. 2. Experimental thrust for the input of Fig. 1.

By observing this figure the following remarks can be made:

- **There is a nonlinear fuel flow/thrust relationship**. In particular, it seems that the nonlinearity induces an increase of the thrust/fuel-flow rate as the fuel-flow increases.
- **The system presents a hysteresis behavior**. This hysteresis characterization has not been previously reported for the thrust and indicates that the system operates over hysteretic loops for the trust dynamic. It is well-known that the characterization of hysteretic loops is a key factor when designing high-performance automatic control systems. Nonetheless, it was found that the hysteresis loop was negligible when the machine was operated with smaller fuel-flow to thrust ranges.

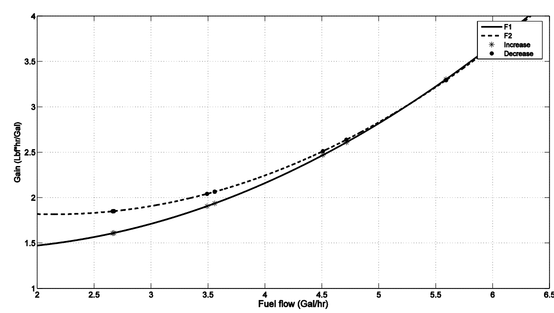

Fig. 3. Static-thrust/fuel-flow relationship.

Using the least squares curve fitting method a nonlinear approximate static-thrust/fuel-flow gain was estimated for each of the hysteresis curves. The resulting approximate functions, denoted as ***F1*** and ***F2*** for the upper and lower curves respectively, are also shown in Fig. 3.

$$\begin{aligned} F1 &= 0.12446521F_f^{\,2} - 0.53414906F_f + 2.388926 \\ F2 &= 0.10462328F_f^{\,2} - 0.28420749F_f + 1.6220854 \end{aligned} \quad (1)$$

Where F_f denotes the input fuel-flow in Gal/hr.

As mentioned before, it was observed that for small-range fuel-flow maneuvers the hysteresis loop is reduced. In those cases it is preferable to use a *mean static-thrust/fuel-flow gain*.

$$F_m = 0.10942528 F_f^{\ 2} - 0.3723699 F_f + 1.9401707 \quad (2)$$

F_m was calculated considering all the data used for calculating the polynomials F_1 and F_2 of eq. (1); i.e. by "mixing" the positive and negative slope data.

IV. DYNAMICS

The dynamical behavior of the thrust response is important when a high-bandwidth automatic control of the air speed is required. In these cases, a feedback controller may operate the turbo reactor in non-static regimes.

A. Signal analysis

The initial experimental data indicated that a precise estimation of the transient characteristics of the process by calculating typical factors, such settling time and over-shoot, would be difficult because of the amount of noise in the measured input/output signals [13]. That is, a direct estimation by using the step response of the system is not adequate due to the measurement conditions. A first step would consist on the use of traditional linear filters which could improve the signal/noise ratio. This may be possible if the noise appears at a frequency rage clearly distinct from the process dynamics [13].

In order to analyze the noise and vibrational frequency ranges, the motor was operated in a constant fuel-flow regime with the maximum allowed sampling frequency (13.15hz). The total thrust was measured (Fig. 4) and its spectrum calculated using the fast Fourier transform (FFT). The low frequency (DC) components of the signal were discarded so that the analysis is focused only on the higher frequency components of the signal. The resulting spectrum, shown in Fig. 5, did not revealed significant high frequency components. Only a low frequency component (around 0.5rad/s) was identified. Therefore, it was not possible to reduce the high-frequency measurement noise and the effects of the vibrations through direct filtering of the measured signal; it is not possible to assess the cut-off frequency of such filter because it would appear that the noise is "almost white" (i.e. present at all frequencies).

B. Least squares identification

Since direct filtering of the measured signal was no possible, it was opted to estimate the plant dynamics using an algorithm which able to reject high levels of noise. In this case the recursive least squares algorithm was chosen for identifying the transient dynamics.

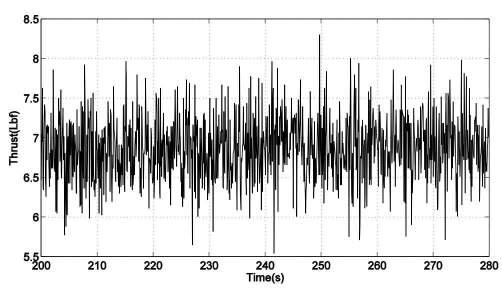

Fig. 4. Measured thrust in the constant fuel-flow experiment.

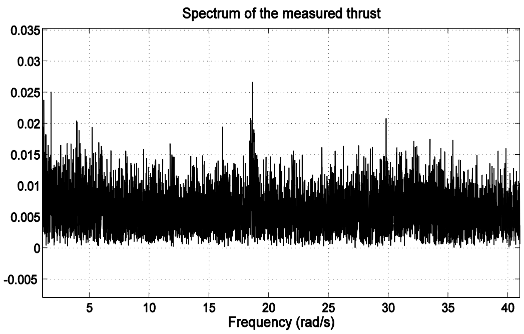

Fig. 5. Spectrum of the measured steady state thrust measurement (low frequency -less than 1rad/s- components omitted).

Let the regression model $y(t) = \theta^T \Phi_{i-1}$ where θ is the parameters vector and Φ the regression vector such that:

$$\begin{aligned} \theta^T &= [\theta_1 \ \ \dots \ \ \theta_{n+m}] \\ \Phi_{i-1} &= [y(i-1) \ \ \dots \ \ y(i-m) \ \ u(i-1) \ \ \dots \ \ u(i-n)]^T \end{aligned} \quad (3)$$

The cost function is defined as:

$$J = \frac{1}{2} \sum_{i=1}^{t} \left[\left(Y_i - \hat{\theta}^T_{i-1} \Phi_{i-1} \right)^2 \right] \quad (4)$$

Where $\hat{\theta}$ is the estimated parameters vector.

Then it can be shown that the estimated parameters which minimize J can be calculated recursively according to [12]:

$$\begin{aligned} \hat{\theta}_i &= \hat{\theta}_{i-1} + F_{i+1} \Phi_{i-1} (Y_i - \hat{\theta}^T_{i-1} \Phi_{i-1}) \\ F_{i+1} &= \left[F_i - \frac{F_i \Phi_{i-1} \Phi^T_{i-1} F_i}{1 + \Phi^T_{i-1} F_i \Phi_{i-1}} \right] \end{aligned} \quad (5)$$

Using this algorithm the discrete transfer function $g(z) = n(z)/d(z)$ can be estimated.

A problem with typical least squares algorithms is that prior knowledge of the structure of the regression model is required. That is, the number of zeros and poles should be known. Although, the zero/pole structure may be known from theoretical models, it is desirable for an experimental identification exercise to identify the possible existence of additional non-modeled dynamics. Therefore, an experimental process for the establishment of an adequate zero/pole structure was devised.

Firstly, a series of zero/pole combinations were identified. In particular all the realizable zero/pole combinations up to 15 poles were identified. That is 1 pole/1 zero, 2 poles/1 zero, 2 poles/2 zeros, 3 poles/1 zero… up to 15 poles/15 zeros. The best zero/pole structure was defined as the one with the less quadratic error. This can be considered as an *auto-constructing* identification scheme. That is, an identification scheme which automatically seeks the best possible model structure such as the *cascade-correlation* algorithm for neural networks [14].

On the other hand, the static thrust identification revealed that the system contains a nonlinear static gain which can interfere with the linear least squares identification process. Therefore, in order to minimize this effect, the least squares identification procedure described in the last paragraph was applied over reduced fuel-flow ranges. These ranges are associated with the **A,B,C,D,E,F** thrust ranges of Fig. 2. Considering the least mean square error of all the ranges, the selected pole/zero structure was 15 poles/1 zero.

Figure 6 shows the Bode plot of the identified discrete transfer functions for each operating range. Two observations can be made. The first simple observation confirms that the steady state gain is nonlinear and depends on the fuel-flow operating range. The second more complicated observation is related to the *over-learning* phenomenon which is commonly found on auto-constructing identification methods such as auto-constructing neural networks [10,11].

The over-learning phenomenon occurs when the auto-constructing identification method incorrectly increases the complexity of the system structure due to the noise present on the measured signals. In this case the 15 poles structure, which had the least quadratic error, is not necessarily related with actual process dynamics. Some of these modes may be related with the high level of noise on the measured signal. Moreover, by analyzing the signal spectrum the bandwidth of the measurement noise could not be properly determined. In Fig. 6 this effect can be observed as a high-frequency gain ripple.

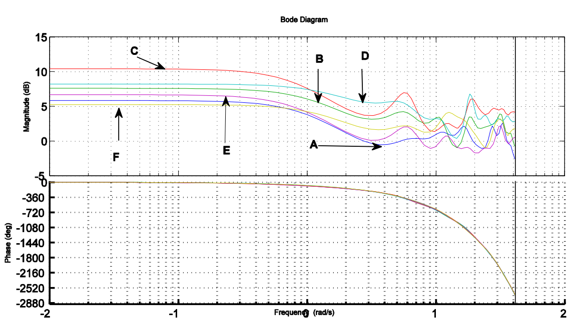

Fig. 6. Bode diagram of the identified discrete transfer functions.

A better interpretation of the problem can be made by observing the pole loci of the identified transfer functions. Fig. 7 shows the poles of all the discrete transfer functions obtained by the least squares identification process.

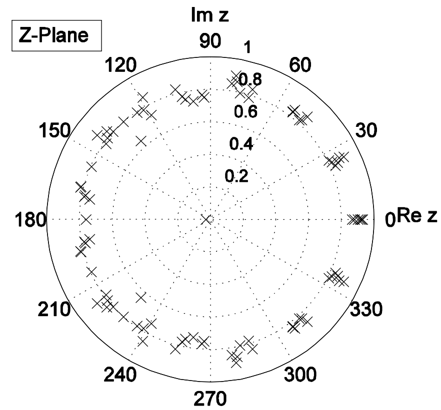

Fig. 7. Root loci of the poles of the identified discrete transfer functions

Figure 7 clearly shows distinct pole groups at low frequencies. It was found that all the transfer functions shared poles in, at least, the first three pole groups which are contained in the set defined by the poles with an argument of less than 30^o and their complex conjugate pair.

This very revealing observation allowed introducing a model reduction akin to a frequency filter (albeit nonlinear). Recall that the argument of discrete poles may be written as $\theta = \omega T$ were ω is the natural damped oscillation frequency in rad/s and T is equal to the sampling period. Therefore, by limiting the maximum allowed pole argument a frequency limit can be imposed.

According the last discussion, a model reduction in which the poles with argument of more than 30^o and their complex conjugate pair were eliminated was applied to all the identified discrete transfer functions. The next figure shows the resulting pole loci of the reduced models.

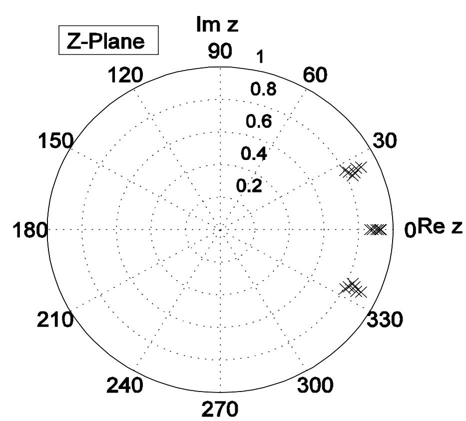

Fig. 8. Root loci of the poles of the reduced discrete transfer functions

The resulting effect of the model reduction can be assessed in Fig. 9 which shows the Bode diagram of the reduced models. These can be directly compared with the original systems (Fig. 5).

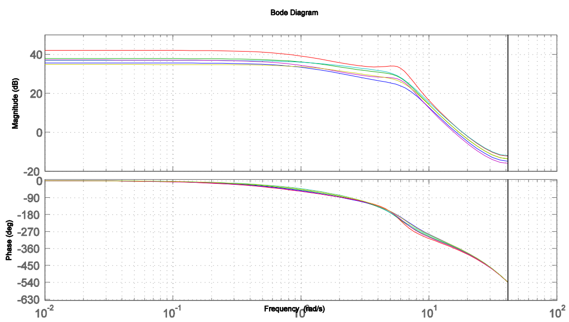

Fig. 9. Bode diagram of the reduced discrete transfer functions.

An additional assessment of the effect of the model reduction is presented in Fig. 10 where the time responses of the original identified transfer functions and the corresponding reduced system are shown. It is clear that the original system has oscillations which are most certainly due to over-learning because of the noise in the measured thrust (see Fig. 6) and a possible time delay. On the other hand the reduced system retains the main establishing time characteristics while eliminating the unwanted oscillations.

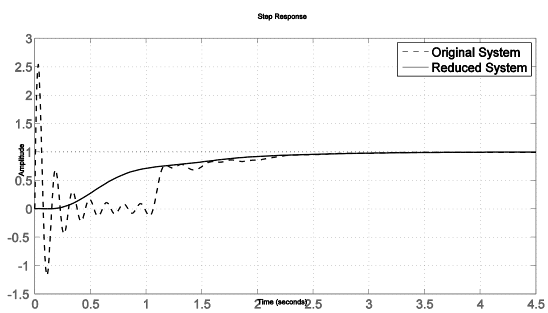

Fig. 10. Step responses of a reduced and non-reduced transfer function

By applying the model reduction method discussed in the last paragraphs, a set of normalized linear systems was derived, these systems (listed in Table I) can be used together with the nonlinear hysteresis gains of eq. (1) as a complete model of the process. The fuel-flow ranges are related with the thrust of Fig. 2. In addition, Table I contains the equivalent continuous-time transfer functions of the identified systems. These transfer functions were calculated considering a zero order hold Z-Transform method.

Finally it is possible to derive a global approximation for the system by calculating the **average** model for all the operating ranges. In addition, the average of the nonlinear hysteresis curves can also be used. This results on the following global model:

$$\frac{\bar{T}_h(s)}{F_f(s)} = \frac{0.11068(s^2 - 36.16s + 459.5)}{(s + 1.3438)(s^2 - 4.322s + 37.85)}$$
$$T_h(t) = F_m(F_f)t_h(t) \tag{6}$$

Where $\bar{T}_h(s)$ is the Laplace transform of $t_h(t)$, $t_h(t)$ is the normalized thrust, $F_m(F_f)$ is eq. (2), $F_f(s)$ is the Laplace transform of the fuel-flow and $T_h(t)$ is the resultant thrust.

Table I

Step	**Fuel Flow Range (Gal/hr)**	**Discrete Transfer Function** T= 0.076s	**Unitary Transfer Function S**
A	2.662-3.492	$\frac{1.068}{59.65z^3 - 140.8z^2 + 118z - 35.71}$	$\frac{7.103s^2 - 252.7s + 3190}{59.65s^3 + 402.7s^2 + 3085s + 3190}$
B	3.492-4.509	$\frac{1.302}{77.32z^3 - 190.1z^2 + 165z - 50.84}$	$\frac{8.025s^2 - 291.1s + 3702}{77.32s^3 + 426.6s^2 + 3163s + 3702}$
C	4.509-5.594	$\frac{1.617}{126.2z^3 - 323z^2 + 293.6z - 95.19}$	$\frac{9.021s^2 - 334.8s + 4306}{126.2s^3 + 468.7s^2 + 4812s + 4306}$
D	5.594-4.715	$\frac{1.51}{69.42z^3 - 166.5z^2 + 140.5z - 41.88}$	$\frac{9.906s^2 - 354.1s + 4473}{69.42s^3 + 461.5s^2 + 3061s + 4473}$
E	4.715-3.573	$\frac{1.014}{70.91z^3 - 177z^2 + 156.9z - 49.83}$	$\frac{5.97s^2 - 218.8s + 2797}{70.91s^3 + 329.1s^2 + 2965s + 2797}$
F	3.573-2.673	$\frac{1.243}{54.51z^3 - 129z^2 + 108.2z - 32.45}$	$\frac{8.278s^2 - 294.6s + 3717}{51.51s^3 + 372.1s^2 + 2683s + 3717}$

V. VALIDATION

The resulting global model (6) represents a compact, yet powerful, model of the gas turbine engine. In order to validate its accuracy the following comparisons were made.

Firstly, the model was validated by comparing the simulated response of system (6) and the stair input experiment. This comparison is shown in Fig. 11. This figure shows a high degree of accuracy on transient and steady state characteristics. Nonetheless a different experiment should be used to asses if the over-learning effect was successfully eliminated.

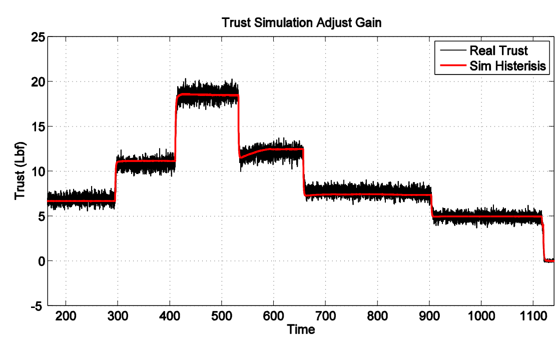

Fig. 11. Comparison between model (6) and the experimental response.

An additional experiment which includes a high degree of transient operations was realized and compared with model (6). The resulting responses are shown in Fig. 12. This figure shows that the global model is highly accurate for all the operating range in both transient and dynamic characteristics.

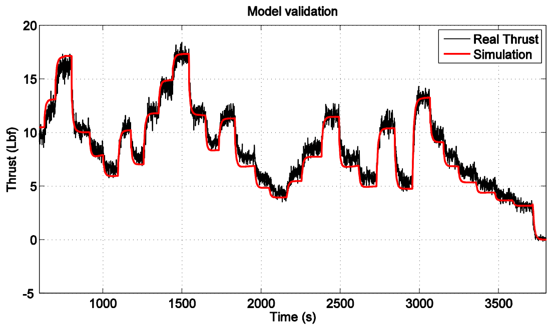

Fig. 12. Comparison between model (6) and the experimental response.

VI. Conclusion

The flow-rate to thrust dynamical model of a laboratory gas-turbo engine was identified and validated using real time experimental data. An identification process which consisted of the estimation of the static and dynamic characteristics of the process was performed.

The static identification consisted on collecting data at several operating ranges. It was observed that the static behavior of the system included a nonlinear hysteresis loop which was also characterized.

The measurement signal was contaminated with high level sensor noise, common to all load cell measurements. The nature of the measurement noise was investigated and it was determined that is was not possible to attenuate the noise with a linear filter.

A series of linear models were estimated using the recursive least squares method. These models were obtained for different operating ranges. The results obtained show that the static gain value depends on the operating condition. A self-constructing algorithm, based on the least square error, was used in order to determine the zero/pole structure of the system.

Further analysis showed that the resulting zero/pole structure was sensible to the measurement noise, which causes the over-learning phenomenon. A method to reduce the order of the identified systems was devised. It was concluded that the fuel-flow rate to thrust dynamical model may be modeled accurately by third order system.

Finally, a nonlinear gain whose value depends on the operating condition, derived from the static hysteresis, can be added to the mean linear system in order to define a simplified *global* approximation.

Further experiments showed that the global approximation gives very accurate prediction of the real thrust both in static and transient regimes.

VII. Acknowledgement

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